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Soliton solutions of the KPII equation

We study the line-soliton solutions of the Kadomtsev-Petviashvili II (KPII) equation, $(-4u_t + u_{xxx} + 6uu_x)_x + 3u_{yy} = 0$. In particular, we characterize the family of elastic $N$-soliton solutions, defined as those solutions for which the number, directions and amplitudes of outgoing solitons (asymptotic line solitons as $y \to \infty$) coincide with those of the incoming solitons (asymptotic line solitons as $y \to -\infty$). This family contains a large variety of solutions; special cases are ordinary soliton solutions, the fully resonant solutions of KPII which also satisfy the finite Toda lattice hierarchy and a family of fully asymmetric soliton solutions. Inbetween these extremes, a large intermediate family of partially resonant solutions exists. We present a decomposition of the whole family of elastic $N$-soliton solutions in terms of equivalence classes of solutions; we further classify these equivalence classes in terms of topologically distinct interaction types; we show how the interaction types are determined by relations among the values of the amplitudes and directions of the individual solitons, and we describe various physical and topological properties of the solutions. The classification of the interaction types decomposes the $2^N$-dimensional parameter space of the soliton amplitudes and directions into $(2^N - 1)!!$ disjoint sectors, each with maximal dimension. Finally, we consider the larger class of line-soliton solutions consisting of arbitrary numbers of incoming and outgoing line solitons and generated by arbitrary combinations of exponential phases, resulting in arbitrary (elastic and inelastic, non-resonant, fully or partially resonant) interaction patterns, and we prove several asymptotic results for the number and types of asymptotic line solitons present.