Integral and integrable algorithms for a nonlinear shallow-water wave equation

An asymptotic higher-order model of wave dynamics in shallow water is examined in a combined analytical and numerical study, with the aim of establishing robust and efficient numerical solution methods. Based on the Hamiltonian structure of the nonlinear equation, an algorithm corresponding to a completely integrable particle lattice is implemented first. Conditions for global existence are identified and $l_1$-norm convergence of the method proved in the limit of zero spatial step size and infinite particles. A fast summation algorithm is introduced to evaluate the integrals of the particle method so that the computational cost is reduced from $O(N^2)$ to $O(N)$, where $N$ is the number of particles. The method possesses some analogies with point vortex methods for 2D Euler equations. In particular, near singular solutions exist and singularities are prevented from occurring in finite time by mechanisms akin to those in the evolution of vortex patches. The second method is based on integro-differential formulations of the equation. These reduce the order of spatial derivatives, thereby relaxing the stability constraint and allowing large time steps in an explicit numerical scheme. In addition to the Cauchy problem on the infinite line, if time permits results on the study of the nonlinear equation posed in the quarter (space-time) plane will be presented.