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Rational solutions of the soliton equations

In this talk I shall discuss special polynomials associated with rational solutions for the second Painlevé equation (PII) and the fourth Painlevé equation (PIV) and associated special polynomials associated with rational solutions of soliton equations which are solvable by the inverse scattering method, including the Korteweg-de Vries equation, the modified Korteweg-de Vries equation and the nonlinear Schrödinger equation.

The Painlevé equations are six nonlinear ordinary differential equations that have been the subject of much interest in the past twenty-five years, which have arisen in a variety of physical applications. Further they may be thought of as nonlinear special functions. Rational solutions of the Painlevé equations are expressible in terms of the logarithmic derivative of certain special polynomials. For PII these polynomials are known as the Yablonskii-Vorob’ev polynomials, first derived in the 1960’s by Yablonskii and Vorob’ev. The locations of the roots of these polynomials is shown to have a highly regular triangular structure in the complex plane. The analogous special polynomials associated with rational solutions of PIV are described and it is shown that their roots also have a highly regular structure and other properties of the polynomials will be discussed.

It is well known that soliton equations have symmetry reductions which reduce them to the Painlevé equations. Hence rational solutions of soliton equations arising from symmetry reductions of the Painlevé equation can be expressed in terms of the aforementioned special polynomials. Also the motion of the poles of the rational solutions of the Korteweg-de Vries equation is described by a constrained Calogero-Moser system describes the motion of the poles of rational solutions of the Korteweg-de Vries equation, as shown by Airault, McKean, and Moser in 1977. The motion of the poles of more general rational solutions of equations in the Korteweg-de Vries and modified Korteweg-de Vries hierarchies, and the motion of zeroes and poles of rational and new rational-oscillatory solutions of the nonlinear Schrödinger equation will be discussed.