Various measures of the growth in complexity of solutions of discrete equations are used to detect integrable cases. Nevanlinna theory is used to find necessary conditions such that there is an admissible finite-order meromorphic solution of the equation $y(z + 1) + y(z - 1) = R(z, y(z))$, where $R$ is rational in $y(z)$ with coefficients meromorphic in $z$. The list of equations satisfying these necessary conditions corresponds to the known discrete Painlevé equations in the class considered. Next, discrete equations are considered over number fields. In this approach the appropriate measure of complexity is the height of the iterates. The height of a rational number $p/q$ is $\max(p, q)$. The first two approaches to discrete integrability are related by Vojta’s dictionary, which connects ideas in Nevanlinna theory to ideas in Diophantine approximation. A new method for showing that the algebraic entropy of many equations is non-zero will be discussed together with connections with singularity confinement. Finally, the extension of this method to so-called ultra-discrete equations will be considered.