An integrable combination of Davey-Stewartson and Kadomtsev-Petviashvili equations

The Davey-Stewartson (DS) equation (the long-wave limit of the Benney-Roskes equation) is given by

\[
\begin{align*}
    i\phi_t + (\sigma_1 \partial_X^2 + \partial_Y^2)\phi &= \sigma_2 |\phi|^2 \phi + 2\sigma_1 \sigma_2 Q\phi, \\
    (+\sigma_1 \partial_X^2 + \partial_Y^2)Q &= -\partial_X^2 |\phi|^2,
\end{align*}
\]

where \( \sigma_1^2 = 1 \).

For the study of soliton solutions of (1) it is useful to write it in Hirota’s bilinear form. Using the substitution

\[
\phi = G/F, \quad Q = 2\sigma_2 \partial_X^2 \log F.
\]

one obtains (after integration) the bilinear form

\[
\begin{align*}
    (iD_t - \sigma_1 D_X^2 + D_Y^2)G \cdot F &= 0, \\
    (+\sigma_1 D_X^2 + D_Y^2) F \cdot F &= -\sigma_2 |G|^2.
\end{align*}
\]

Here we consider the DSII variant (\( \sigma_1 = 1 \)) and then, after a further 45 degree rotation, we get

\[
\begin{align*}
    (iD_t + 2D_x D_y)G \cdot F &= 0, \\
    (D_x^2 + D_y^2) F \cdot F &= -\sigma_2 |G|^2.
\end{align*}
\]

In [1] we found the following integrable generalization of (2)

\[
\begin{align*}
    (-2iD_t + 3D_x D_y + i\alpha D_x^3 + c)G \cdot F &= 0, \\
    \left[bD_x^2 + a(\alpha^2 D_x^4 - 3D_y^2 + 4\alpha D_x D_t)\right] F \cdot F &= 2|G|^2,
\end{align*}
\]

with arbitrary parameters \( a, b, c, \alpha \). If \( \alpha = c = 0 \) we recover DSII (2), while for \( \alpha = 1, b = 0, G = 0 \) we get the Kadomtsev-Petviashvili equation (the KPI variant) in bilinear form. Thus (3) is a combination of the two most important \((2+1)\)-dimensional equations, but the combination only seems to work for the DSII and KPI variants.

In our contribution we will discuss the solutions of system (3) and its nonlinear form.